Department of Mathematics and Computer Science

Friday, January 25, 2019, 4:10 pm

COLLOQUIUM TALK

Speaker: Gregory Galperin (EIU)

Old Main 2231

Two Remarkable Constructions in Hyperbolic Plane $\mathbb{H}^2$ and Their Justification in the Klein Model $\mathbb{K}^2$

Abstract:

In hyperbolic geometry $\mathbb{H}^2$, there is a very special and unique correspondence $x \leftrightarrow \Pi(x)$ between a segment of length x and the respective acute angle $\Pi(x)$ called "*the angle of parallelism*." In my talk, I will give solutions to the following two construction problems:

I *Given segment x, construct the angle of parallelism* $\Pi(x)$; and the inverse problem:

II *Given an acute angle φ, construct a segment x whose angle of parallelism $\Pi(x)$ equals φ.*

Both constructions must be done in the hyperbolic plane $\mathbb{H}^2$ by "hyperbolic" compass and "hyperbolic" straightedge.

The first construction belongs to *Janos Bolyai*, one of the creators of hyperbolic geometry. It is very elegant; however, to justify his construction, Bolyai attracts non-elementary tough results from the solid hyperbolic geometry $\mathbb{H}^3$.

The second construction is based partially on the Bolyai's construction and partially on a theorem by american mathematician *George Martin*. A known proof of Martin's theorem is purely hyperbolic and is based on the Bolyai-Lobachevsky formula for the angle of parallelism and for hyperbolic trigonometry.

I will justify both constructions and prove the Martin theorem in the disc Klein model $\mathbb{K}^2$ of the hyperbolic plane $\mathbb{H}^2$. Both of my proofs are very simple and are based on elementary Euclidean geometry and basic Euclidean trigonometry.

All the necessary terms: *the angle of parallelism* $\Pi(x)$, *the Klein model* $\mathbb{K}^2$, *the measurement of distances in the Klein model*, etc., will be introduced and explained during the talk.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)