Department of Mathematics and Computer Science

Friday, October 6, 2017, 4:10 pm

COLLOQUIUM TALK

Speaker: Joseph Vandehey (OSU)

Old Main 2231

Normality vs. Determinism

Abstract:

Suppose a real number $x \in [0, 1)$ has base-b expansions $0.a_1a_2a_3\ldots$. We say this number is normal (in base b) if every finite string $s = [d_1, d_2, \ldots, d_k]$ of base-b digits appears in the expansion of x with limiting frequency b^{-k}. While many constructed examples of normal numbers are known, the overall set of normal numbers is quite mysterious, and we still don't know if π, e, $\sqrt{2}$, or indeed any commonly used mathematical constant is normal to any base.

D.D. Wall showed that if $0.a_1a_2a_3\ldots$ is normal, then so is $0.a_ka_{k+\ell}a_{k+2\ell}\ldots$ for any $k, \ell \in \mathbb{N}$. In other words, selecting along arithmetic progressions preserves normality. After Furstenberg, Weiss, and Kamae connected normality to the disjointness of dynamical systems, it was shown that selecting along any deterministic sequence (of positive density) preserves normality. In fact, only deterministic sequences preserve normality.

This leads to a number of natural questions.

1) What IS a deterministic sequence, anyway?

2) What are some examples of deterministic sequences?

3) Is the fact that selecting along deterministic sequences preserves normality a deep result?

4) What if we want to select along a sequence that is dependent on our normal number?

5) What applications does this theory have?

Parts of this talk are joint work with V. Bergelson (OSU).

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)