Department of Mathematics and Computer Science

Friday, October 18, 2019, 4:10 pm

COLLOQUIUM TALK

Speaker: Gregory Galperin (EIU)

Old Main 2210

A hidden symmetry in the sequences "*First Digits of* 2^n, 3^n, 4^n, 5^n, ..."

Abstract: The sequence of the **last** (rightmost) digits of the integers 2^n is **periodic**:

(2, 4, 8, 6), (2, 4, 8, 6), ...

On the contrary, the sequence of the **first** (leading) digits of the integers 2^n is **chaotic**:

2, 4, 8, 1, 3, 6, 1, 5, ...

QUESTION: *Does this chaotic sequence of the* **first digits** *contain the digit 7 or the digit 9? If yes, then which digit, 7, 8, or 9, appears more frequently in this sequence?*

The same question can be posed for the sequences "*First digits of* 3^n, *of* 4^n, *of* 5^n, ...". Or, generally, we can consider sequences of more than one leading digit and ask similar questions:

• *Can* 2^n *and* 3^n *both have one or more leading digits of the number* $\pi = 3.1415\ldots$, *or both have one or more leading digits of the number* $e = 2.71828\ldots$*?*

• *Can* 2^n *and* 5^n *both start with the same one or more leading digits and if yes, then what could these digits be? The same question for the triplet* 2^n, 3^n, 5^n.

It turns out that to answer to that kind of pure arithmetic questions, one needs to take into consideration special **dynamical systems** on a **circle** S^1 and on a **torus** $\mathbb{T}^2$.

In my talk, I will associate the above sequences with dynamical systems on the circle and on the torus, and will also tell about my recent discovery of a **hidden symmetry** in the set of the exponents $\{\mathbf{n}\}$ for the sequences 2^n, 3^n, 4^n, 5^n, ...

All the core ideas will be explained on a very elementary level, so everyone, especially students, are very welcome to attend the talk.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)