Department of Mathematics and Computer Science

Friday, October 26, 2018, 4:10 pm

COLLOQUIUM TALK

Speaker: Gregory Galperin (EIU)

Old Main 2231

Non-Convex Polyhedra inscribed in a sphere, and a New Intriguing 4-dimensional Polytope

Abstract. If a polygon is inscribed in a circle, it must be convex; if a polygon is circumscribed around a circle, it also must be convex. Its easy to formulate the same statement for 3-dimensional polyhedra inscribed in or circumscribed around a 2-dimensional sphere.

However, a proof of this spacial statement similar to the one for the inscribed polygons fails, and its not immediate to find a flaw in the respective proof. It turns out that any such proof cannot be correct: it turns out that

there exists a non-convex solid polyhedron with 5 vertices inscribed in a sphere!

In my talk I will show how to construct such a polyhedron P_5 and then prove a general statement for **a family of <u>non-convex</u> polyhedra P_n with a prescribed number vertices $n > 4$ situated on the surface of a sphere**.

Then I will discuss non-convex polyhedra *circumscribed around a sphere.*

The remaining part of my talk will be devoted to a description of a very intriguing and exotic **convex 4-dimensional polytope Q_n^4 with a prescribed number $n \geq 5$ of vertices which <u>has no diagonals</u> inside it**.

The **4D-polytope Q_n^4 has exactly $\binom{n}{2} = n(n-1)/2$ edges** which means that any segment containing arbitrary two vertices of the polytope Q_n^4 must be <u>an edge</u> of this polytope.

In the 3D-space such a polyhedron exists only if $n = 4$, i.e. for simplexes only!

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)