

Friday, November 4, 2016, 4:10 pm

COLLOQUIUM TALK

Speaker: Evgeny Gordon (EIU)

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What Is Nonstandard Analysis and Why It Can Become the Analysis of the Future

Abstract:

The creators of Calculus, Newton and Leibniz, had different points of view on the notion of an infinitesimal. According to Newton, an infinitesimal is a changing quantity that becomes smaller and smaller and eventually disappears in the process of its change. According to Leibniz, a (positive) infinitesimal is an ideal positive number that is smaller, than any quantifiable positive number. Mathematicians of the 18th century freely used both approaches to the infinitesimals. However, in the beginning of the 19th century mathematicians encountered some paradoxes that involved the basic notions of calculus. This forced them to develop the logically rigorous foundations of Calculus. It was done on the basis of Newton's approach, and for many years mathematicians believed in impossibility to do this on the basis of Leibniz's approach. Only in the late sixties of the last century Abraham Robinson introduced Leibniz's infinitesimals in Calculus at the same level of rigor as it had been done before for the Newton's infinitesimals. Robinson gave a lame name "Nonstandard Analysis" to the Calculus with constant infinitesimals that he constructed.

Nonstandard Analysis makes it possible to consider as absolutely rigorous definitions the intuitive informal formulations of basic definitions of Calculus, such as "A function is continuous, if an infinitesimal change of an independent variable implies an infinitesimal change of a function". This has simplified many proofs in Calculus, made them more intuitive, and even allowed to obtain some new results in functional analysis, probability theory and some other areas of mathematics. Many mathematicians (especially logicians) in the beginning were very enthusiastic by opening perspectives. After the Robinson's talk at the Institute for Advanced Study (Princeton) in March 1973, the greatest logician in the world Kurt Gödel even made the following statement "... there are good reasons to believe that non-standard analysis in some version or other, will be the analysis of the future."

During more than fifty years after the first publication on nonstandard analysis, Gödel's prediction has not been realized yet. However, I do not lose hope. In this talk I will present a new form of Nonstandard Analysis, in which, in my opinion, it may become the analysis of the future, and I will try to justify this my hope.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)
